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RESEARCH MEMORANDUM

AN ANALYTICAL STUDY OF SIDESLIP ANGLES AND VERTICAL-TAIL
LOADS IN ROLLING PULLOUTS AS AFFECTED BY SOME
CHARACTERISTICS OF MODERN HIGH-SPEED
AIRPLANE CONFIGURATIONS

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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

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SUMMARY

The rolling-pullout maneuver has been shown to be pertinent and possibly critical to the design of the vertical tail with regard to the loads produced on it. The trends in airplane configurations and mass distributions have been such as to emphasize the critical nature of the maneuver, and more exact calculations of the maneuver for adequate estimations of the vertical-tail load appear to be required. Because extensive studies of the maneuver are not available and necessary calculations of such maneuvers depend on the stability and control derivatives used, an analytical study was undertaken to assess the significance of various yawing-moment stability derivatives on the sideslip angles attained in rolling pullouts. Some effects of oscillations in the sideslip angle during a rolling pullout on the variation of the vertical-tail load were also studied.

The results of this investigation indicate that the directional-stability derivative $C_{n\beta}$ is the most critical of the yawing-moment stability derivatives with regard to an adequate estimation of the vertical-tail load. The adverse yawing-moment coefficient due to aileron deflection $(C_{n\delta_a})$, the yawing-moment coefficient due to rolling (C_{nr}) , and the coefficient of the damping in yaw (C_{nr}) are less critical to an accurate estimation of the vertical-tail load than $C_{n\beta}$ in the order given. For airplane configurations and mass distributions different than those studied, some of these derivatives may be more critical, and expressions for estimating the effects of possible changes or errors in these

derivatives on the maximum angle of sideslip are included. As has been noted in the past, the mass distribution of modern high-speed airplanes has an important effect on the maximum sideslip angles attained in rolling pullouts and its influence cannot be neglected. In rolling pullouts an oscillation in the sideslip angle which is influenced by piloting techniques is set up and peaks in the oscillation of sideslip angle subsequent to the first may cause larger vertical-tail loads than the first.

INTRODUCTION

The rolling-pullout maneuver (any maneuver in which rolls occur during high g flight conditions) has been shown to be pertinent to design considerations from the standpoint of the loads produced on the vertical tail (refs. 1 to 3). The more recent work related to this flight condition (ref. 3) has indicated that for modern high-speed airplanes, particularly those with mass distributed primarily along the fuselage, the simplified expressions used in the past (ref. 2) for adequately estimating the maximum sideslip angle in a rolling pullout generally are not applicable and that more complete methods of estimating the sideslip angle and vertical-tail load are necessary.

Recently some concern has been indicated as to the degree of accuracy needed in estimating or measuring the stability and control derivatives used for calculating rolling-pullout maneuvers so that the variations in sideslip angle and vertical-tail load may be estimated with sufficient accuracy. In order to understand this problem better, the results of an analytical study of the effects of large variations of some of the lateral-stability-derivative coefficients on the maximum angle of sideslip at the first peak of its oscillation are presented in this paper. Also, expressions for estimating the effects of small variations of or errors in these coefficients on the maximum sideslip angle have been developed and are presented. This analytical study was made for a modern high-speed airplane in a 6g rolling-pullout maneuver. Three different mass distributions were considered in the study.

Some concern has also been indicated recently regarding the oscillations in the sideslip angle and as to whether larger sideslip angles and vertical-tail loads might be encountered in such oscillations subsequent to the time of the first peak. Although the problem seems to be closely associated with piloting techniques used the results of a study of some factors influencing these oscillations are also included in this paper.

COEFFICIENTS AND SYMBOLS

The motions presented herein were calculated primarily about the stability system of axes. A diagram of these axes showing positive directions of the forces and moments is presented in figure 1.

C_L	lift coefficient, $\frac{L}{\frac{1}{2}\rho V^2 S}$
C_Y	lateral-force coefficient, $\frac{Y}{\frac{1}{2}\rho V^2 S}$
C_l	rolling-moment coefficient, $\frac{L'}{\frac{1}{2}\rho V^2 S b}$
C_m	pitching-moment coefficient, $\frac{M}{\frac{1}{2}\rho V^2 S \bar{c}}$
C_n	yawing-moment coefficient, $\frac{N}{\frac{1}{2}\rho V^2 S b}$
$C_{l_{\delta_a}}$	increment of rolling-moment coefficient caused by aileron deflection
$C_{n_{\delta_a}}$	increment of yawing-moment coefficient caused by aileron deflection
L	lift, nW, lb
Y	lateral force, lb
L'	rolling moment, ft-lb
M	pitching moment, ft-lb
N	yawing moment, ft-lb
S	wing area, sq ft
b	wing span, ft
ρ	air density, slugs/cu ft

V	velocity, ft/sec
$I_{X_0}, I_{Y_0}, I_{Z_0}$	moments of inertia about X, Y, and Z principal axes, respectively, slug-ft ²
I_X, I_Y, I_Z	moments of inertia about X, Y, and Z stability axes, respectively, slug-ft ²
I_{XZ}	product of inertia (positive when principal axis is inclined above flight path), slug-ft ²
μ	relative density coefficient based on span, $m/\rho S b$
m	mass of airplane, W/g , slugs
W	weight of airplane, lb
g	acceleration due to gravity, 32.2 ft/sec ²
n	normal acceleration divided by acceleration due to gravity
α	angle of attack $\left(\tan^{-1} \frac{w}{u} \text{ in body system of axes} \right)$, deg
β	angle of sideslip, $\sin^{-1} \frac{v}{V}$, radians except when otherwise noted
η	inclination of principal longitudinal axis of inertia with respect to flight path, deg
u, v, w	components of velocity V along the X, Y, and Z body axes, respectively; v is also component of V along Y stability axis, ft/sec
θ	angle of pitch, radians
ψ	angle of yaw, radians
ϕ	angle of roll, radians except when otherwise noted
$\dot{\phi}$ or p	rolling angular velocity, radians/sec
$\dot{\theta}$ or q	pitching angular velocity, radians/sec
$\dot{\psi}$ or r	yawing angular velocity, radians/sec
$\dot{\beta}$	rate of change of angle of sideslip with time

$\ddot{\phi}$ rate of change of rolling angular velocity with time

$\ddot{\psi}$ rate of change of yawing angular velocity with time

$$C_{l_{\beta}} = \frac{\partial C_l}{\partial \beta}$$

$$C_{n_{\beta}} = \frac{\partial C_n}{\partial \beta}$$

$$C_{Y_{\beta}} = \frac{\partial C_Y}{\partial \beta}$$

$$C_{l_p} = \frac{\partial C_l}{\partial \frac{pb}{2V}}$$

$$C_{n_p} = \frac{\partial C_n}{\partial \frac{pb}{2V}}$$

$$C_{l_r} = \frac{\partial C_l}{\partial \frac{rb}{2V}}$$

$$C_{n_r} = \frac{\partial C_n}{\partial \frac{rb}{2V}}$$

METHODS OF ANALYSIS

Analysis of Influence of Stability Derivatives

This analysis was based on a treatment of the three linear lateral equations of motion. The effects of cross coupling of inertia moments which occur in rolling motions during pitching maneuvers, however, were included by assuming a constant pitching velocity to exist for any one calculation. By assuming a constant pitching velocity the cross-coupled

inertia moments, as influenced by pitching in the lateral equations, could be included in the lateral equations and these equations would still be linear. The cross-coupled inertia terms appearing in the yawing and rolling equations are, respectively,

$$(I_X - I_Y) \ddot{\theta} \dot{\phi}$$

and

$$(I_Y - I_Z) \ddot{\theta} \dot{\psi}$$

These terms are thus only functions of $\dot{\phi}$ and $\dot{\psi}$ (assuming constant $\dot{\theta}$) and were included as additions to the C_{n_p} and C_{l_r} terms in the linear lateral equations. These equations so modified are

$$I_X \ddot{\phi} + I_{XZ} \ddot{\psi} - \left[C_{l_\beta} \beta + C_{l_p} \frac{\dot{\phi} b}{2V} + \left(C_{l_r} + \frac{I_Y - I_Z}{\frac{\rho}{2} V^2 S b} \frac{2V\dot{\theta}}{b} \right) \frac{\dot{\psi} b}{2V} + C_{l_{\delta_a}} \delta_a \right] \frac{\rho}{2} V^2 S b = 0 \quad (1a)$$

$$I_Z \ddot{\psi} + I_{XZ} \ddot{\phi} - \left[C_{n_\beta} \beta + \left(C_{n_p} + \frac{I_X - I_Y}{\frac{\rho}{2} V^2 S b} \frac{2V\dot{\theta}}{b} \right) \frac{\dot{\phi} b}{2V} + C_{n_r} \frac{\dot{\psi} b}{2V} + C_{n_{\delta_a}} \delta_a \right] \frac{\rho}{2} V^2 S b = 0 \quad (1b)$$

$$mV(\dot{\beta} + \dot{\psi}) - \left(\frac{C_L}{n} \phi + C_{Y_\beta} \beta \right) \frac{\rho}{2} V^2 S = 0 \quad (1c)$$

As is evidenced in reference 3, however, the pitching velocity generally is not constant, particularly in rolling pullouts. The method outlined above, therefore, is only indicative of the effects of cross coupling of inertia terms and does not give precise results.

In equation (1c) the term $\frac{C_L}{n} \frac{\rho}{2} V^2 S$ represents the weight of the airplane and ϕ is a substitution for $\sin \phi$ necessary for linearization of the equations. This later substitution common to all lateral-stability work is only approximate for the work treated in this paper where the angle of roll ϕ at the time of maximum sideslip angle β is of the order of from 40° to 60°. In addition it is assumed that the aerodynamic

derivatives expressed as partials are adequate for the large motions experienced during the rolling-pullout maneuvers.

Analysis of Effects of Oscillation

This analysis was based partly on a calculation of a 6g rolling pullout by the use of a step-by-step procedure presented in reference 3. For this calculation it was also believed that a variation in velocity or dynamic pressure should be considered in that an airplane would be slowing down because of the high drag associated with high g flight. This change in velocity was estimated on the basis that a deceleration existed proportional to the increment of drag between 1g and 6g flight. The velocity change was based on the equation

$$m \frac{dV}{dt} + \Delta C_D \frac{1}{2} \rho V^2 S = 0$$

from which

$$V = \frac{2mV_0}{\Delta C_D \rho S V_0 t + 2m}$$

where V_0 is the initial velocity.

Additional analysis was based on the linear lateral equations of motion previously discussed (eqs. (1)), which were used to make calculations for several different constant pitching velocities.

CALCULATIONS

The calculations presented in this paper are for a modern high-speed type of airplane having characteristics the same as those of airplane A of reference 3. For all conditions calculated by the use of equations (1), a constant angle of attack of 13° was used. For the calculations to determine the effects of variations in some of the stability derivatives, the airplane was assumed to be in a 6g pullout, as previously noted, at the time of rolling and a corresponding constant pitching velocity of 0.179 radian per second was used. The calculations were made to determine the value of the sideslip angle at the first peak of its oscillation. Three mass distributions were used, one in which the mass was heavily distributed along the fuselage $\left(\frac{I_{Z_0}}{I_{X_0}} = 12 \right)$, one simulating the

mass distributions common in the past $\left(\frac{I_{Z_0}}{I_{X_0}} = 3\right)$, and an intermediate value $\left(\frac{I_{Z_0}}{I_{X_0}} = 6\right)$. The stability and control coefficients used for the calculations are presented in table I. The mass characteristics for the three mass distributions are presented in table II. The effects of varying the stability-derivative coefficients C_{n_p} , $C_{n_{\delta_a}}$, $C_{n_{\beta}}$, and C_{n_r} were determined for each of the three mass distributions. The variations used for these coefficients are presented in table I. These derivatives are those which have been considered generally important to the sideslip angle in rolling maneuvers in the past (ref. 2). When calculating the effects of varying each of these coefficients, all the others were held constant at their basic values.

In order to check on the possibility of having larger vertical-tail loads at peaks in an oscillation in the sideslip angle subsequent to the first, consideration was given to results presented in reference 3. These results (for a 6g rolling pullout) indicate a rapidly increasing value of sideslip angle subsequent to the first peak in β and at the end of a 90° roll. Since a pilot might roll further than 90°, these calculations were extended past the second peak in the oscillation of β . As was previously noted, consideration of a possible change in velocity was also made. This calculation was based on the airplane basic condition listed in table I and for the mass distribution where $\frac{I_{Z_0}}{I_{X_0}} = 12$ (table II).

In order to understand better the oscillations in sideslip angle and how they are affected by pitching velocity as exists in a pull-out or high g maneuver, the effects of increasing the pitching velocity $\dot{\theta}$ in equation (1) were also investigated. Pitching velocities of from 0 to 0.716 radian/sec were investigated. The conditions for these calculations were for the airplane having the basic coefficients listed

in table I and for the mass distribution where $\frac{I_{Z_0}}{I_{X_0}} = 12$.

RESULTS AND DISCUSSION

Some General Considerations of Sideslip in Rolling Maneuvers

Since the mass distribution of modern airplanes has been shown to have an important effect on the sideslip angle in rolling maneuvers (ref. 3) and the mass distribution was apparently of only secondary importance for airplanes of World War II type with regard to sideslip in rolling maneuvers (refs. 1 and 2), some simple general considerations concerning the variations in sideslip are presented herein to aid in understanding the problem before the results of this paper are presented. Qualitative variations in sideslip for different airplane conditions are shown in figure 2. If in the absence of other aerodynamic forces and moments a rolling moment was applied about the X stability axis and if all moments of inertia were very large except for the rolling moment of inertia (I_{X_0}) , the airplane would roll about its X principal axis such that the variation in β would be approximately $\eta \sin \phi$ and at 90° of roll the angle of sideslip would be equal to η (the original angle of attack of the principal axis), whereas this original angle of attack would go to zero. This fact, however, is only part of the problem in that aerodynamic yawing moments are also acting. If the airplane had no directional stability $(C_{n\beta} = 0)$ but had adverse yawing moments such as exist from aileron deflections $(C_{n\delta_a} \delta_a)$, rolling $(C_{n\dot{\phi}} \frac{\dot{\phi}}{2V})$, and cross-coupled inertia moments $[(I_X - I_Y)\dot{\phi}]$, increments in β would be added to the variation, $\eta \sin \phi$ such that the upper curve of figure 2 would be obtained. Directional stability $(C_{n\beta})$ will reduce the sideslip angle from that given by the upper curve and cause an oscillation in β . Increasing amounts of directional stability reduce the amount of sideslip at the first peak as shown in figure 2. The first peak in β occurs at a time equal to about 1/2 the period of the natural lateral oscillation of the airplane. Increasing values of directional stability $(C_{n\beta})$ reduce this time so that the first peak in β occurs at a time when less possible ultimate β exists; in addition, more yawing acceleration to reduce β prior to this time is provided.

As previously noted, upon application of a rolling moment about the X stability axis and in the absence of other aerodynamic forces and moments, the variation of the sideslip angle would approximate $\eta \sin \phi$ only if the moments of inertia I_{Z_0} and I_{Y_0} were very large relative to I_{X_0} , and if the angle of attack went to zero at 90° of roll. If,

however, the pilot maintained a constant angle of attack during the rolling maneuver, then for the conditions of an applied rolling moment about the X stability axis, in the absence of other aerodynamic forces and moments, as previously discussed, the variation of β would be approximately $\frac{I_{XZ}}{I_Z} \phi$ (see appendix A) rather than $\eta \sin \phi$. This would be the case for any mass distribution even where I_{Z_0} and I_{Y_0} were not large relative to I_{X_0} . Under these conditions, the effects of the yawing moments just discussed with relation to the variation $\eta \sin \phi$ would be added in the same sense to $\frac{I_{XZ}}{I_Z} \phi$. Variations of $\frac{I_{XZ}}{I_Z} \phi$ for the mass-distribution conditions treated in this paper are shown in figure 3.

Presentation of Results

The results of the calculations showing the effects on the angle of sideslip, at the first peak, of variations in the yawing-stability derivatives are shown in figures 4 to 7. The effects of changes in the directional-stability derivative $C_{n\beta}$ with relation to some general considerations previously discussed are shown in figure 8.

The results of the calculations which were extended to show the effects of oscillations in sideslip are shown in figure 9. The results of calculations showing the effects of increasing values of pitching velocity on the lateral oscillations as affected by cross-coupled inertia moments are shown in figures 10 and 11.

Effects of Variation of Some Lateral-Stability Coefficients

For the convenience of the reader who may find it necessary to make estimations of stability derivatives for calculating rolling-pullout maneuvers and vertical-tail loads, a list of references (refs. 4 to 16) is included. These references indicate procedures for estimating derivatives for both subsonic and supersonic conditions and include experimental results for some recent measurements of directional-stability derivatives at supersonic speeds.

Effect of C_{n_p} .— The effects on the maximum angle of sideslip shown for an extreme variation in C_{n_p} (0 to -0.26) are not large for the

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configuration and mass distributions investigated (fig. 4). The incremental effects of C_{n_p} are about the same for each of the three mass distributions (approximately $2/3^\circ$ for the range of C_{n_p} studied); however as a percent of the total sideslip angle the effects become larger as the mass distribution changes from $\frac{I_{Z_0}}{I_{X_0}} = 12$ to 3. In calculating rolling motions to estimate the sideslip angle and vertical-tail load an estimation or measurement of C_{n_p} , therefore, is more critical for cases where the mass is distributed more evenly along the wings and fuselage than for cases where the mass is distributed primarily along the fuselage. In addition previous studies (ref. 2) for World War II type airplanes, for which the mass distributions were such that I_{Z_0}/I_{X_0} was of the order of 3 or less and for which the wing had a more dominant influence on the stability derivatives than do the low-aspect-ratio wings of current airplanes, indicated that C_{n_p} had an even more important influence on the maximum sideslip angle than for the conditions presented herein.

An expression for use in evaluating the effects of possible errors in C_{n_p}

$$\Delta\beta = - \frac{\rho V S b^2}{4 I_Z} \Delta C_{n_p} \beta_{\max} \frac{t_{\beta_{\max}}}{2}$$

is developed in appendix A and should be applicable to the general case. An application of the expression to the cases shown in figure 4 gives increments in β almost the same as are given by the complete solutions of figure 4. The effects of very large increments in C_{n_p} , as might be introduced artificially for auxiliary damping of lateral motions (see ref. 17), cannot be estimated by this simple expression.

Effect of $C_{n_{\delta_a}}$. The effects on the maximum sideslip angle of a relatively large variation in $C_{n_{\delta_a}}$ (0 to -0.0070) became larger as the mass distribution was changed from $\frac{I_{Z_0}}{I_{X_0}} = 12$ to 3 (fig. 5). The total increment for the entire variation of $C_{n_{\delta_a}}$ represented about 20 percent and 70 percent maximum β (at $C_{n_{\delta_a}} = 0$) for the mass

distributions when I_{Z_0}/I_{X_0} was 12 and 3, respectively. Thus estimations or measurements of $C_{n_{\delta_a}}$ are more critical in estimating the vertical-tail load for mass distributions where the weight is distributed along both the wing and fuselage than for mass distributions where the weight is primarily along the fuselage. For World War II type airplanes $C_{n_{\delta_a}}$ has been shown to have an even more important influence on maximum β (ref. 2) than is shown for the case where $\frac{I_{Z_0}}{I_{X_0}} = 3$.

The expression developed in appendix A

$$\Delta\beta = - \frac{\rho V^2 S b}{2 I_Z} \Delta C_{n_{\delta_a}} \frac{(t_{\beta_{\max}})^2}{2}$$

is applicable to the general case for estimating possible effects of errors in $C_{n_{\delta_a}}$. An application of the expression to the cases shown

in figure 5 gives a conservative estimate of a possible error, when this error is in a direction to increase β_{\max} .

Effects of C_{n_r} .- The damping derivative C_{n_r} has been shown not to be an important factor in previous simplified expressions for estimating the maximum sideslip angle in rolling maneuvers (refs. 1 and 2). The results shown in figure 6 of the effects of large variations in C_{n_r} substantiate this point. Variations of C_{n_r} from -1.090 to -0.2725 did not appreciably affect the sideslip angle for any of the mass distributions. An expression for estimating possible effects of errors in C_{n_r} is included in appendix A. This study was not sufficiently extensive to cover the possible case of large damping obtained by artificial means which might be used for some airplanes. The effects of C_{n_r} may not be small for such cases.

Effects of C_{n_p} .- Changes in the directional-stability derivative C_{n_p} had a very large effect on the maximum sideslip angle for all mass distributions (fig. 7). This result is in agreement with previous studies

(refs. 2 and 3) which indicate a dominant influence of C_{n_β} . Variations in C_{n_β} were most critical for the mass distribution for which $\frac{I_{Z_0}}{I_{X_0}} = 3$.

For this mass distribution, reducing C_{n_β} to one-fourth its value increased the maximum sideslip angle by $2\frac{1}{2}$ times. For the mass distribution where $\frac{I_{Z_0}}{I_{X_0}} = 12$, the maximum sideslip angle was increased by only 65 percent.

The maximum sideslip angle, therefore, is not inversely proportional to C_{n_β} as previous simplified expressions (ref. 2) might indicate. The

proximity to inverse proportionality becomes less as mass is distributed more along the fuselage. Thus, larger values of directional stability, although leading to smaller sideslip angles, do not necessarily lead to smaller vertical-tail loads in rolling maneuvers. For example, the airplane used in these calculations has unstable wing-fuselage directional stability $C_{n_\beta} = -0.0015$. If in reducing C_{n_β} from 0.0065 to 0.0016

the assumption is made that only the area of the tail is changed in proportion to the change in the required contribution to directional stability of the tail, then the load carried on the normal tail ($C_{n_\beta} = 0.0065$)

at maximum β (5.08°) when the mass is distributed mainly along the fuselage ($\frac{I_{Z_0}}{I_{X_0}} = 12$) will be about $1\frac{1}{2}$ times the load carried on a small tail ($C_{n_\beta} = 0.0016$) at its maximum β (8.37°) for the same mass distribution.

If on the other hand the maximum sideslip angles were inversely proportional to the directional-stability derivative C_{n_β} , as they tended

to be for World War II type airplanes and mass distributions, the large tail, for this case, would have carried a load of about 65 percent of that of the small tail.

Because of the very dominant role of C_{n_β} no attempt has been made to develop an expression to determine the effect of possible errors in it. Relatively high accuracy in measuring or estimating C_{n_β} appear

essential to a reasonable estimation of maximum sideslip angle and the vertical-tail load.

Figure 8 shows the variations of β , through its maximum value, with angle of roll ϕ (where ϕ is a time variable) in relation to the variation $\frac{I_{XZ}}{I_Z} \phi$ and $\eta \sin \phi$ which were discussed previously as concepts of the variation of β based on the inertia characteristics of the airplane. The relations of the variation of β with $\frac{I_{XZ}}{I_Z} \phi$ for each of the three mass distributions, each with three values of C_{n_β} , are similar; whereas the variations of β with relation to $\eta \sin \phi$ change progressively as I_Z/I_{X_0} is decreased from 12 to 3.

Effect of mass distribution.— The most noticeable effect shown in figures 4 to 7 is that of mass distribution. For the condition of large directional stability ($C_{n_\beta} = 0.0065$), in figures 4 to 6 and for one point in figure 7, redistributing the weight from a value of the ratio $\frac{I_Z}{I_{X_0}} = 3$ to a value of 12 nearly doubles the maximum sideslip angle and, thus, the vertical-tail load. This effect of loading is not included in the simplified expressions of references 1 and 2 and is a primary reason why these expressions do not adequately estimate the maximum sideslip angle for modern high-speed airplanes.

Effects of Oscillation in Sideslip Angle on Vertical-Tail Load

The results of the calculations shown in figure 9 indicate that the vertical-tail load can be larger at the second peak of an oscillation in the sideslip angle than at the first peak during a rolling pull-out. The results show a 20-percent increase in the sideslip angle and an approximate 10-percent decrease in dynamic pressure with a resulting increase in vertical-tail load of 10 percent. For these results the elevator was held fixed throughout the maneuver. In the variation in velocity shown, no consideration is made of possible changes in thrust or changes in gravitational force along the wind axis which might occur as the airplane reorients itself in space. The variation in velocity may therefore be unconservative. The angle of attack at the time of the second peak in β was somewhat larger than the angle of attack at the

start of the motion. This factor causes an increase in the sideslip angle. If a pilot attempted to maintain a constant angle of attack, forward motion of the stick would have been required at the time of the second peak in β . The reduced angle of attack and any changes in pitching velocity associated with forward stick motion would, of course, influence the magnitude of the sideslip angle at the second peak. No method exists, however, of evaluating what the piloting techniques in a rolling pullout will be, and thus whether the first or subsequent peaks in β are critical to the vertical-tail load depends on the individual case. It is believed that the effects of changes in the various stability derivatives previously discussed relative to the first peak in the oscillation in β would also be applicable to subsequent peaks, but no proof of this belief has been attempted.

Because of the possibility of larger vertical-tail loads at times in a rolling-pullout maneuver subsequent to the time of the first peak in β , a study of the effects of changes in pitching velocity on the oscillations of the sideslip angle was made. The results (fig. 10) show that the effect of increasing the pitching velocity is to increase the sideslip angle at the first peak and reduce the stability (fig. 11). This result is in agreement with results of reference 3 which showed that the inclusion of the effects of pitching increased β at the first peak. Positive stability exists for all pitching velocities except for the largest pitching velocity ($\dot{\theta} = 0.716$ radian/sec). The sideslip angle is larger at the second peak, however, for the case where $\dot{\theta} = 0.537$ radians/sec even though the airplane does have positive stability. This is primarily the result of the non-oscillatory part of β (fig. 10(b)) which increases positively with time when pitching exists. The variations in β which would exist if β was merely oscillating about zero are shown in figure 10(c) and indicate reduced values of β at the second peak except for the unstable case. It is conceivable that if the pitching velocity increased with time the variation of β with time would tend to shift from one curve to another in figure 10(a) with the relative possibility of larger sideslip angles at the second peak. If the angle of attack was also allowed to increase with time the second peak could be larger by the mere expedient of $\eta \sin \phi$ or

$$\frac{I_{XZ}}{I_Z} \phi \text{ being larger than if the angle of attack remained constant.}$$

The 6g pullout condition treated in this paper has a pitching velocity of 0.179 radian/sec at the beginning of the motion. It should be pointed out however that the pitching velocity $\dot{\theta}$ is not uniquely defined by the normal acceleration in a rolling-pullout maneuver and it is possible that much larger values can exist when rolling and sideslip are present. For the rolling-pullout case treated in reference 3

the pitching velocity (initially 0.179 radian/sec) reached a maximum of about 0.44 radian/sec. A brief explanation of this effect is given in appendix B.

CONCLUSIONS

An analytical investigation of the effects of variations of the yawing-moment coefficients on the maximum sideslip angle during rolling pullouts and the effects of oscillations of the sideslip angle on the maximum vertical-tail load indicates the following conclusions:

1. The directional-stability derivative C_{n_β} is the most critical of the yawing-moment stability derivatives with regard to an adequate estimation of the vertical-tail load. The adverse yawing-moment coefficient due to aileron deflection $(C_{n_{\delta_a}})$, the yawing-moment coefficient due to rolling (C_{n_p}) , and the coefficient of the damping in yaw (C_{n_r}) are less critical to an accurate estimation of the vertical-tail load than C_{n_β} in the order given.
2. For airplane configurations and mass distributions different than those studied, some of these derivatives may be more critical, and expressions for estimating the effects of possible changes or errors in these derivatives on the maximum angle for the general case are included.
3. As has been indicated in the past, the mass distribution of modern high-speed airplanes has an important effect on the maximum sideslip angles attained in rolling pullouts and its influence cannot be neglected.
4. In rolling pullouts an oscillation in the sideslip angle which is influenced by piloting techniques is set up. As a result of this condition, peaks in the oscillation of sideslip angle subsequent to the first may cause larger vertical-tail loads than the first.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
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APPENDIX A

DERIVATION OF EXPRESSIONS FOR ESTIMATING EFFECTS OF DERIVATIVES

The calculations presented herein are for a limited number of conditions and the results may not be generally applicable to every airplane configuration or condition. Accordingly simple expressions have been developed to determine the possible effects of errors in some of the lateral-stability-derivative coefficients on the maximum sideslip angle for the general case. The development of these expressions is based on the assumption that variation of sideslip angle β with time has been calculated by complete methods and therefore that the maximum sideslip angle and the time to reach this angle are known. With this information known, the effects on the maximum sideslip angle of possible errors in the derivatives used to calculate the motion can be estimated approximately by these expressions. Certain limitations regarding these expressions are also given.

From equation 1(c), by dividing by mV , rearranging, and integrating a value of β may be written as follows:

$$\beta = \int_0^t \left[\left(\frac{C_L}{n} \dot{\phi} + C_{Y\beta} \beta \right) \frac{\rho V^2 S}{2mV} - \dot{\psi} \right] dt \quad (A1)$$

where from equation 1(b) (by similar procedures),

$$\dot{\psi} = \int_0^t \left[\left(C_{n\beta} \beta + C_{n_p} \frac{\dot{\phi}}{2V} + C_{n_r} \frac{\dot{\psi}}{2V} + C_{n_{\delta_a}} \delta_a \right) \frac{\rho V^2 S b}{2I_Z} - \frac{I_{XZ}}{I_Z} \ddot{\phi} + \frac{I_X - I_Y}{I_Z} \dot{\phi} \right] dt \quad (A2)$$

Increments in β due to $\dot{\psi}$ in equation (A1) might therefore be obtained approximately by substitution of equation (A2) into equation (A1) as follows:

$$\Delta\beta(\dot{\psi}) = - \int_0^t \int_0^t \left[\left(C_{n\beta} \beta + C_{n_p} \frac{\dot{\phi}}{2V} + C_{n_r} \frac{\dot{\psi}}{2V} + C_{n_{\delta_a}} \delta_a \right) \frac{\rho V^2 S b}{2I_Z} - \frac{I_{XZ}}{I_Z} \ddot{\phi} + \frac{I_X - I_Y}{I_Z} \dot{\phi} \right] dt \, dt \quad (A3)$$

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which can be broken into component parts as, for example,

$$\Delta\beta(C_{n_p}) = - \int_0^t \int_0^t C_{n_p} \frac{b}{2V} \frac{\rho V^2 S b}{2I_Z} \ddot{\phi} dt dt \quad (A4)$$

$$\Delta\beta(C_{n_{\delta_a}} \delta_a) = - \int_0^t \int_0^t C_{n_{\delta_a}} \delta_a \frac{\rho V^2 S b}{2I_Z} dt dt \quad (A5)$$

Now if the limits of these integrations are the time at which β_{\max} occurs, increments in β , relative to β_{\max} , will be obtained. Further if it is presumed that some error in C_{n_p} and $C_{n_{\delta_a}}$ exists (ΔC_{n_p} and $\Delta C_{n_{\delta_a}}$), then the effect of this error on the maximum value of β in terms of an estimated increment in β ($\Delta\beta(\Delta C_{n_p})$ and $\Delta\beta(\Delta C_{n_{\delta_a}} \delta_a)$) can be obtained by the following equations:

$$\Delta\beta(\Delta C_{n_p}) = - \frac{\rho S b^2 V}{4I_Z} \Delta C_{n_p} \int_0^{t_{\beta_{\max}}} \int_0^t \frac{d\ddot{\phi}}{dt} dt dt \quad (A6)$$

and

$$\Delta\beta(\Delta C_{n_{\delta_a}} \delta_a) = - \frac{\rho S b V^2}{2I_Z} \Delta C_{n_{\delta_a}} \delta_a \int_0^{t_{\beta_{\max}}} \int_0^t dt dt \quad (A7)$$

where $t_{\beta_{\max}}$ is the time at which maximum β (at the first peak) occurs. In equation (A7), δ_a is considered to be a constant and is thus removed from under the integral sign, because it is assumed that the ailerons are instantaneously set to a fixed deflection at zero time.

Solution of equation (A6) requires an expression for $\frac{d\ddot{\phi}}{dt}$. Equation 1(a) indicates that $\frac{d^2\phi}{dt^2}$ and therefore $\frac{d\ddot{\phi}}{dt}$ is a rather complicated

function. The integral

$$\int_0^t \int_0^t \frac{d\phi}{dt} dt dt = \int_0^t \int_0^t d\phi dt$$

however is proportional to the area under a curve of ϕ against time. Thus an approximation giving the best variation of ϕ with time to that originally obtained could be used. If as an approximation the rolling angular acceleration was assumed to be constant, then

$$\frac{d^2\phi}{dt^2} = K \quad \frac{d\phi}{dt} = Kt \quad \phi = \frac{Kt^2}{2}$$

and

$$\int_0^t \int_0^t \frac{d\phi}{dt} dt dt = K \int_0^t \int_0^t t dt dt = \frac{Kt^3}{6}$$

With this assumption the definite integral $\int_0^t \int_0^t \frac{d\phi}{dt} dt dt$ becomes

$$\int_0^t \int_0^t \frac{d\phi}{dt} dt dt = \phi \frac{t}{3}$$

and if limits of 0 and $t_{\beta_{\max}}$ are used, then equation (A6) is

$$\Delta\beta (\Delta C_{n_p}) = - \frac{\rho S b^2 V}{4 I_Z} \Delta C_{n_p} \phi_{\beta_{\max}} \frac{t_{\beta_{\max}}}{3} \quad (A8)$$

and integration of equation (A7) gives

$$\Delta\beta (\Delta C_{n_{\delta_a}} \delta_a) = - \frac{\rho S b^2 V}{2 I_Z} \Delta C_{n_{\delta_a}} \delta_a \frac{(t_{\beta_{\max}})^2}{2} \quad (A9)$$

Equations (A8) and (A9) are the simple expressions presented in the body of this paper for use in evaluating the effects on β_{\max} of

possible errors in C_{n_p} and $C_{n_{\delta_a}}$. If the variation of ϕ used in integrating equation (A6) to obtain equation (A8) $\phi = \frac{Kt^2}{2}$ does not closely represent the variation actually obtained, some other variation such as $\phi = Kt$ might be considered or the actual area under a curve of ϕ against time could be used.

This approach may also be used to evaluate the effects of other factors in equation (A3) in order to determine the effects of changes in I_{XZ} , $I_X - I_Y$, and C_{n_r} . These effects may be expressed as

$$\Delta\beta(\Delta I_{XZ}) = \int_0^{t_{\beta_{\max}}} \int_0^t \frac{\Delta I_{XZ}}{I_Z} \ddot{\phi} dt dt = \frac{\Delta I_{XZ}}{I_Z} \phi_{\beta_{\max}} \quad (A10)$$

$$\Delta\beta[\Delta(I_X - I_Y)] = - \int_0^{t_{\beta_{\max}}} \int_0^t \frac{\Delta(I_X - I_Y)}{I_Z} \ddot{\phi} dt dt$$

and if the same variation in ϕ as was used in equation (A8) is assumed,

$$\Delta\beta[\Delta(I_X - I_Y)] = - \frac{\Delta(I_X - I_Y)}{I_Z} \ddot{\phi}_{\beta_{\max}} \frac{t_{\beta_{\max}}^3}{3} \quad (A11)$$

which would be true only if $\ddot{\phi}$ was approximately constant and when the variations in ϕ are subject to the same conditions indicated for equation (A8). Also,

$$\Delta\beta(\Delta C_{n_r}) = - \Delta C_{n_r} \frac{\rho S b^2 V}{4 I_Z} \int_0^{t_{\beta_{\max}}} \int_0^t \frac{d\psi}{dt} dt dt$$

and if a variation in ψ of a form similar to that assumed for ϕ is assumed and subject to the same limitations, the following equation is obtained:

$$\Delta\beta(\Delta C_{n_r}) = - \Delta C_{n_r} \frac{\rho S b^2 V}{4 I_Z} \psi_{\beta_{\max}} \frac{t_{\beta_{\max}}^3}{3} \quad (A12)$$

All increments presented presume that a motion has been calculated by accurate methods and therefore that $\phi_{\beta_{\max}}$, $t_{\beta_{\max}}$, and β_{\max} are known. It is further presumed that small changes in the derivatives, which would represent possible errors, have an approximately linear effect on the maximum sideslip angle. The conditions presented herein (figs. 4 to 6) indicate such an approximate linear effect for even relatively large changes in C_{n_p} , $C_{n_{\delta_a}}$, and C_{n_r} . It should be noted,

however, that if the increments in the various derivatives considered are such as to cause appreciable changes in the period of the oscillatory motion of β and if in the expressions presented where integrals of ϕ or its derivatives occur, the incremental change in the particular derivative under consideration (as for example C_{n_p}) has an appreciable effect

on the variation of ϕ with time, the expressions lose significance. Such changes in the variation of ϕ would occur probably only if the real root of the characteristic equation denoting the damping in roll were appreciably affected by the incremental change in the derivative (see ref. 17). In any event it is believed that equations (A8) to (A12) may be used to give an approximate quantitative effect of reasonable errors in these derivatives. No attempt has as yet been made to evaluate the effects of possible errors in the stability derivatives relating to the rolling moment.

APPENDIX B

EFFECT OF SIDESLIP AND ROLLING ON THE PITCHING VELOCITY

The fundamental equation for the normal force is (B1)

$$F_Z = m(\dot{w} - uq + vp)$$

where u , v , and w are components of the velocity along the x , y , and z axes, respectively, q is the pitching velocity, and p is the rolling velocity. In the stability system of axes,

$$w = \dot{w} = 0$$

$$u = V \cos \beta$$

$$v = V \sin \beta$$

and

$$q = \dot{\gamma}$$

where $\dot{\gamma}$ is the rate of change of the flight-path angle. The force F_Z in the stability system of axes is the sum of the aerodynamic lift L and the weight component along the Z -axis. Equation (B1) thus becomes

$$L - W_Z = m(V \cos \beta \dot{\gamma} - V \sin \beta p) \quad (B2)$$

If β is assumed to be approximately equal to $\sin \beta$ and $\cos \beta$ is assumed to be approximately 1, equation (B2) becomes

$$\dot{\gamma} = \frac{L - W_Z}{mV} + \beta p \quad (B3)$$

The sum of the angle of attack and flight-path angle equals the angle of pitch, and if the angle of attack is held constant, $\dot{\theta} = \dot{\gamma}$, so that $\dot{\theta}$, the pitching velocity, is equal to equation (B3). Thus any positive value of βp adds a centrifugal force to the lift which causes an increase in the pitching velocity even though the angle of attack and lift are held constant.

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TABLE I.- COEFFICIENTS EMPLOYED FOR CALCULATIONS

[Aerodynamic characteristics are referred to stability axes]

	Basic	Modified	
Wing area, sq ft	166.5		
Wing span, ft	22.7		
C_{l_β} , per deg	-0.0032		
C_{n_β} , per deg	0.0065	0.00325	0.001625
C_{Y_β} , per deg	-0.015		
C_{l_p} , per radian	-0.225		
C_{n_p} , per radian	-0.130	0	-0.26
C_{l_r} , per radian	0.235		
C_{n_r} , per radian	-1.090	-0.545	-0.2725
$C_{l_{\delta_a}}$	0.0197		
$C_{n_{\delta_a}}$	-0.0035	0	-0.0070
C_L	0.78		



TABLE II.- MASS CHARACTERISTICS

Loading	Weight, lb	Airplane relative density coefficient, μ	Moments of inertia about principal axes, slug-ft ²			Moments and products of inertia, about stability axes, slug-ft ²				$\frac{I_{Z_0}}{I_{X_0}}$
			I_{X_0}	I_{Y_0}	I_{Z_0}	(a)				
			I_{X_0}	I_{Y_0}	I_{Z_0}	I_X	I_Y	I_Z	I_{XZ}	
1	20,828	71.9	5,381	63,971	65,559	8,435	63,971	62,550	13,200	12
2	20,828	71.9	10,950	58,402	65,559	13,760	58,402	63,050	11,970	6
3	20,828	71.9	21,900	47,452	65,559	24,080	47,452	63,480	9,580	3

^aAngle of attack assumed constant at 13°; body and principal axes were assumed to coincide.

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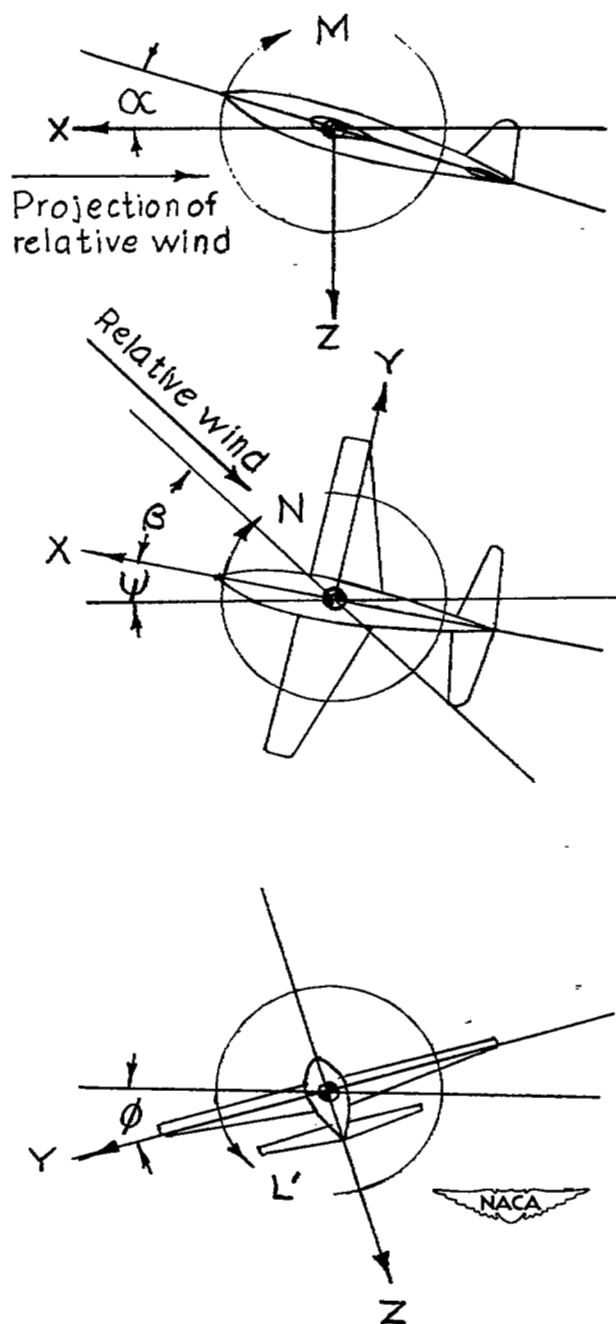


Figure 1.- Sketch depicting the stability system of axes. Each view presents a plane of the axes system as viewed along the third axis.

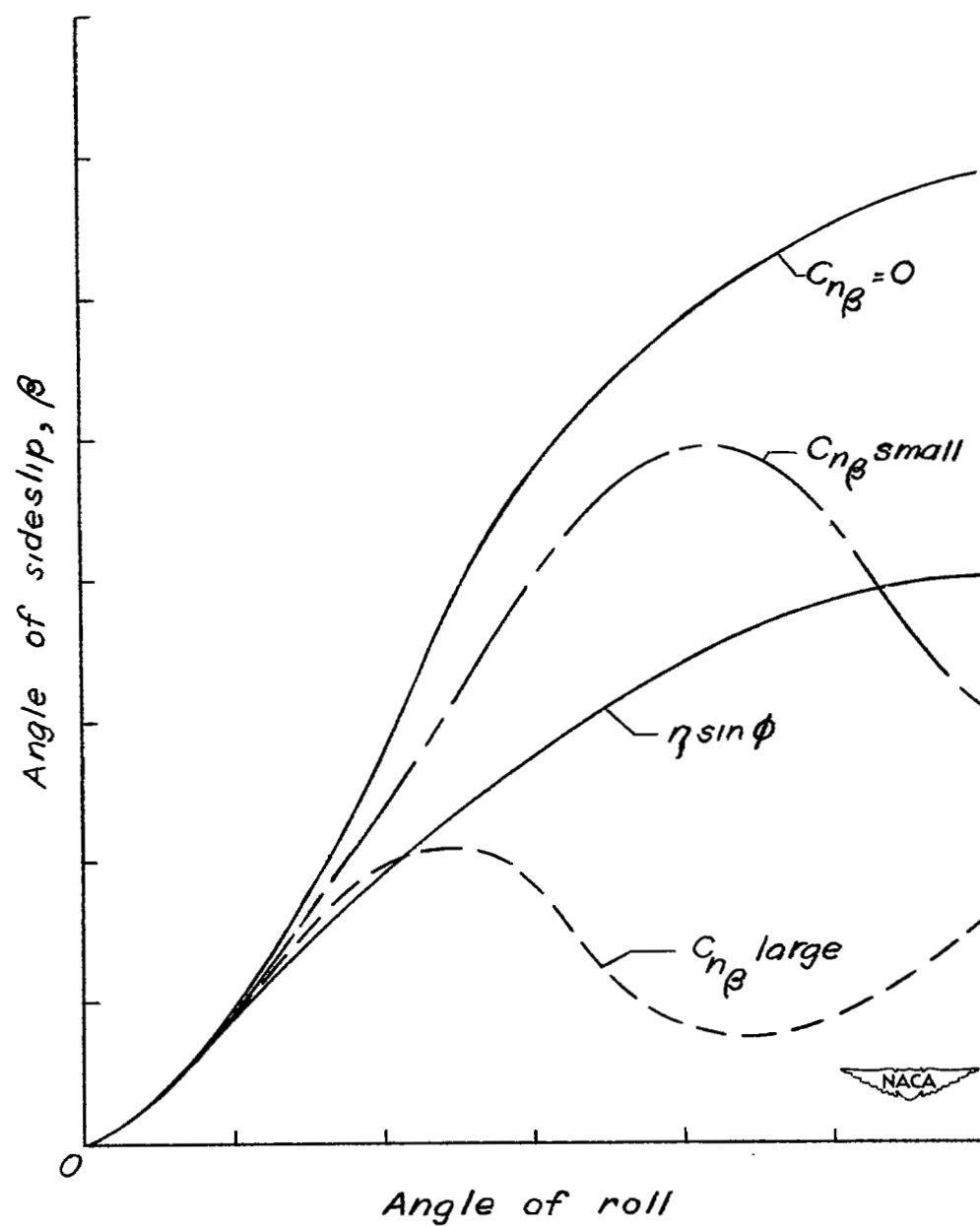


Figure 2.- Qualitative variations in angle of sideslip for different airplane conditions.

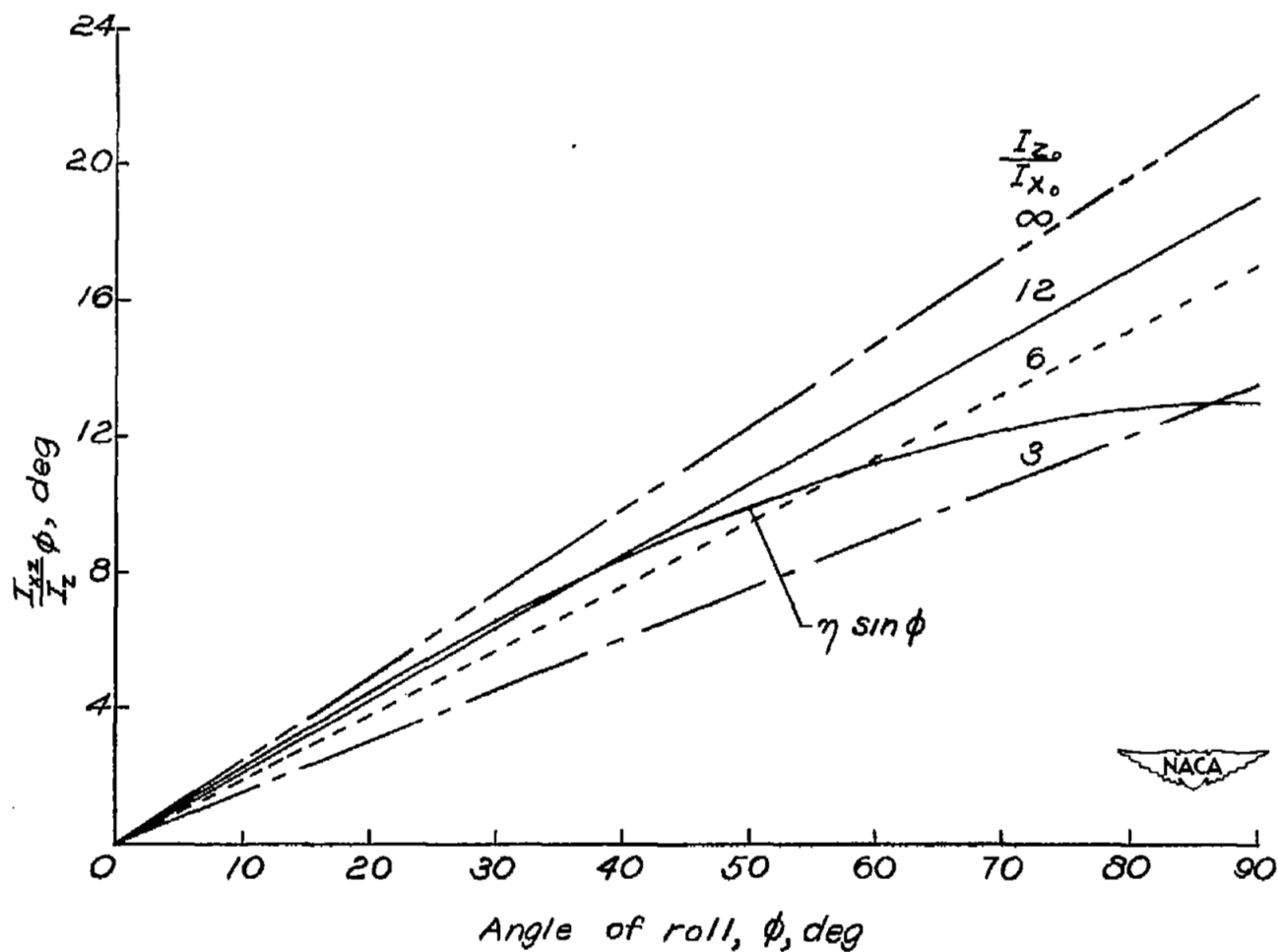


Figure 3.- Variations of $\frac{I_{xz}}{I_z} \phi$ with ϕ for a constant angle of attack of 13° for the mass distributions considered. Curve of $\eta \sin \phi$ included for comparison.

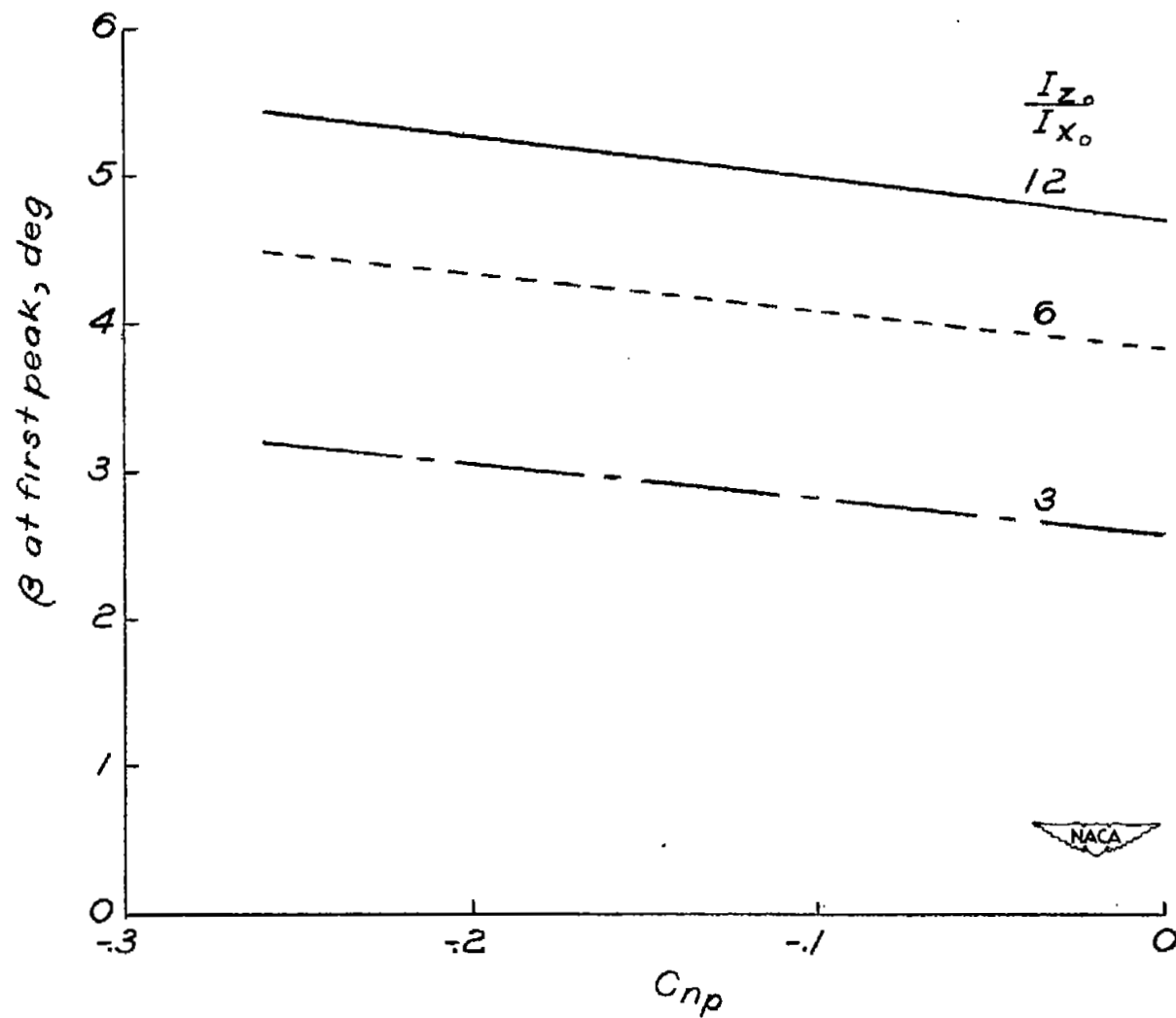


Figure 4.- Effect of variations in C_{np} on the angle of sideslip at the first peak of its oscillation.

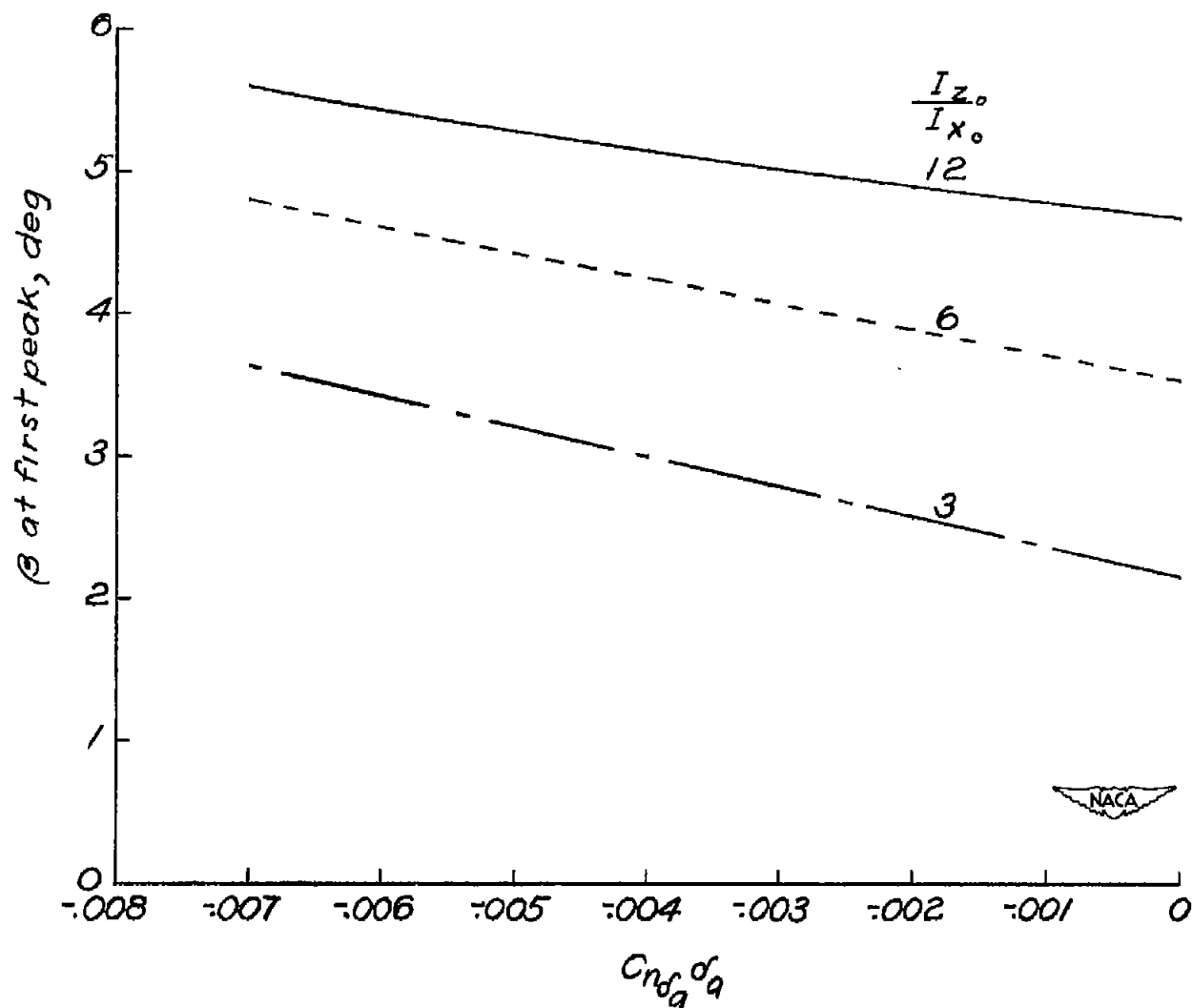


Figure 5.- Effect of variations in $C_{n\delta_a}\delta_a$ on the angle of sideslip at the first peak of its oscillation.

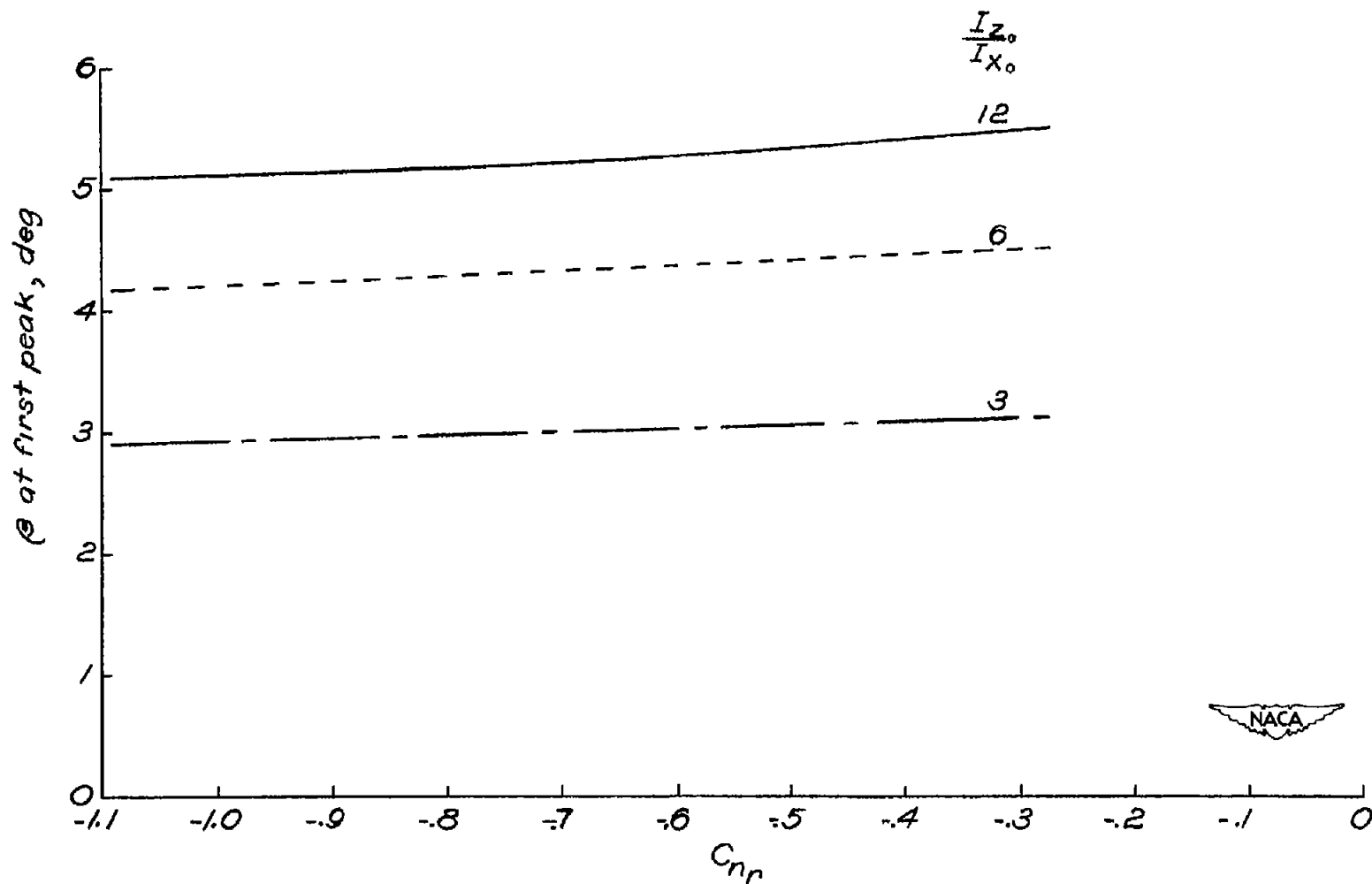


Figure 6.- Effect of variations in C_{nr} on the angle of sideslip at the first peak of its oscillation.

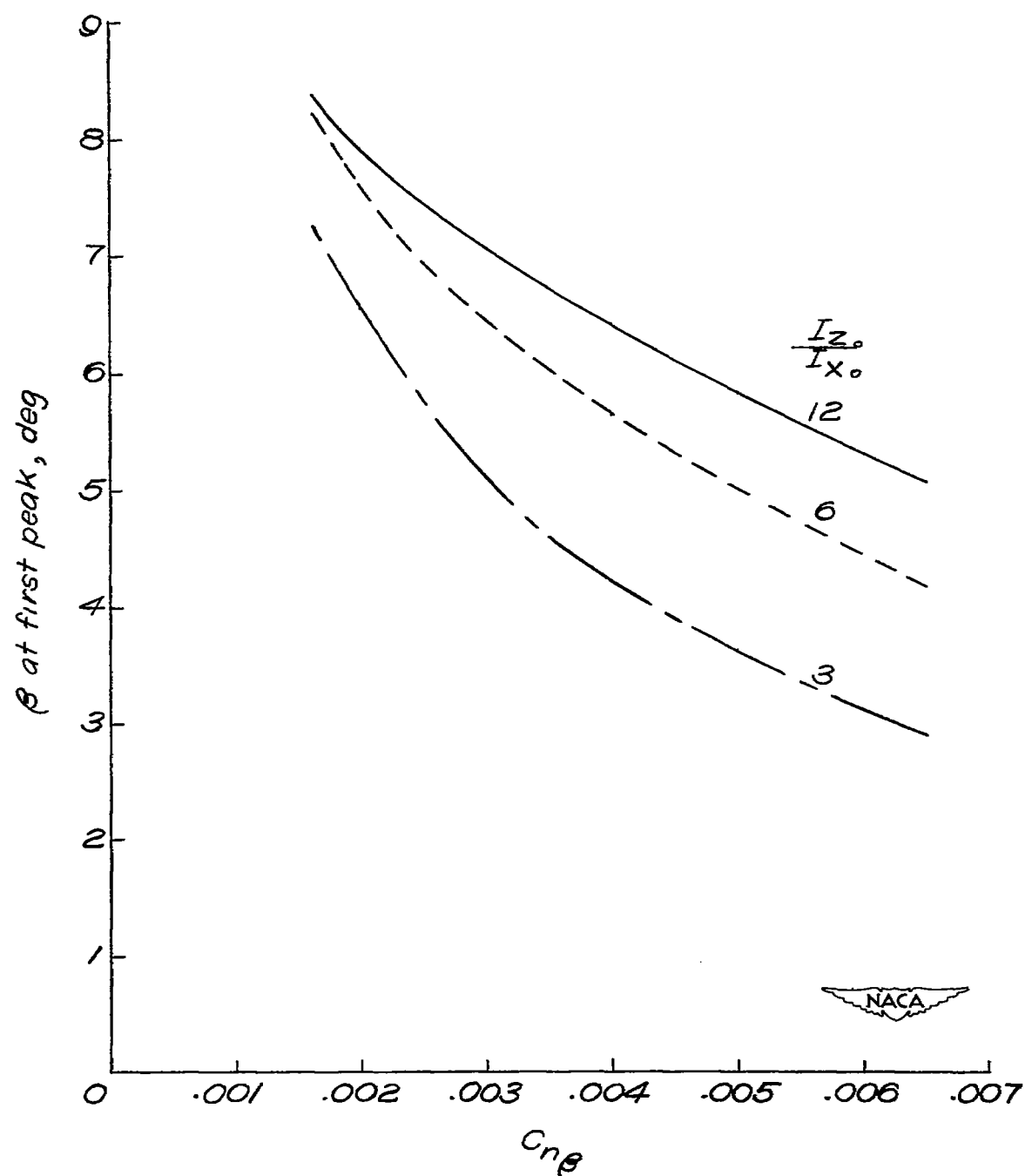


Figure 7.- Effect of variations in $C_{n\beta}$ on the angle of sideslip at the first peak of its oscillation.

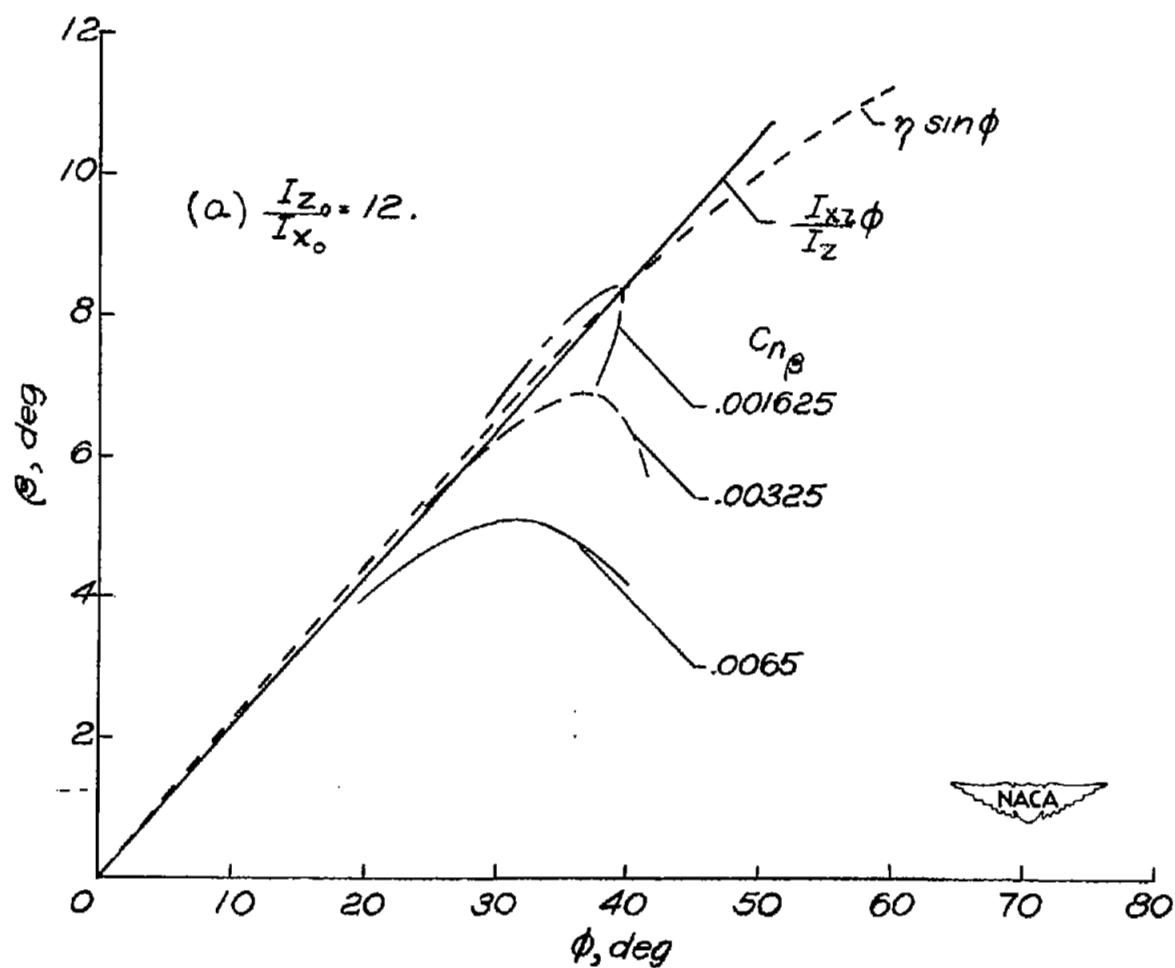


Figure 8.- A comparison of the variation of the angle of sideslip near the first peak of its oscillation with the variations of $\frac{I_{xz}}{I_z} \phi$ and $\eta \sin \phi$.

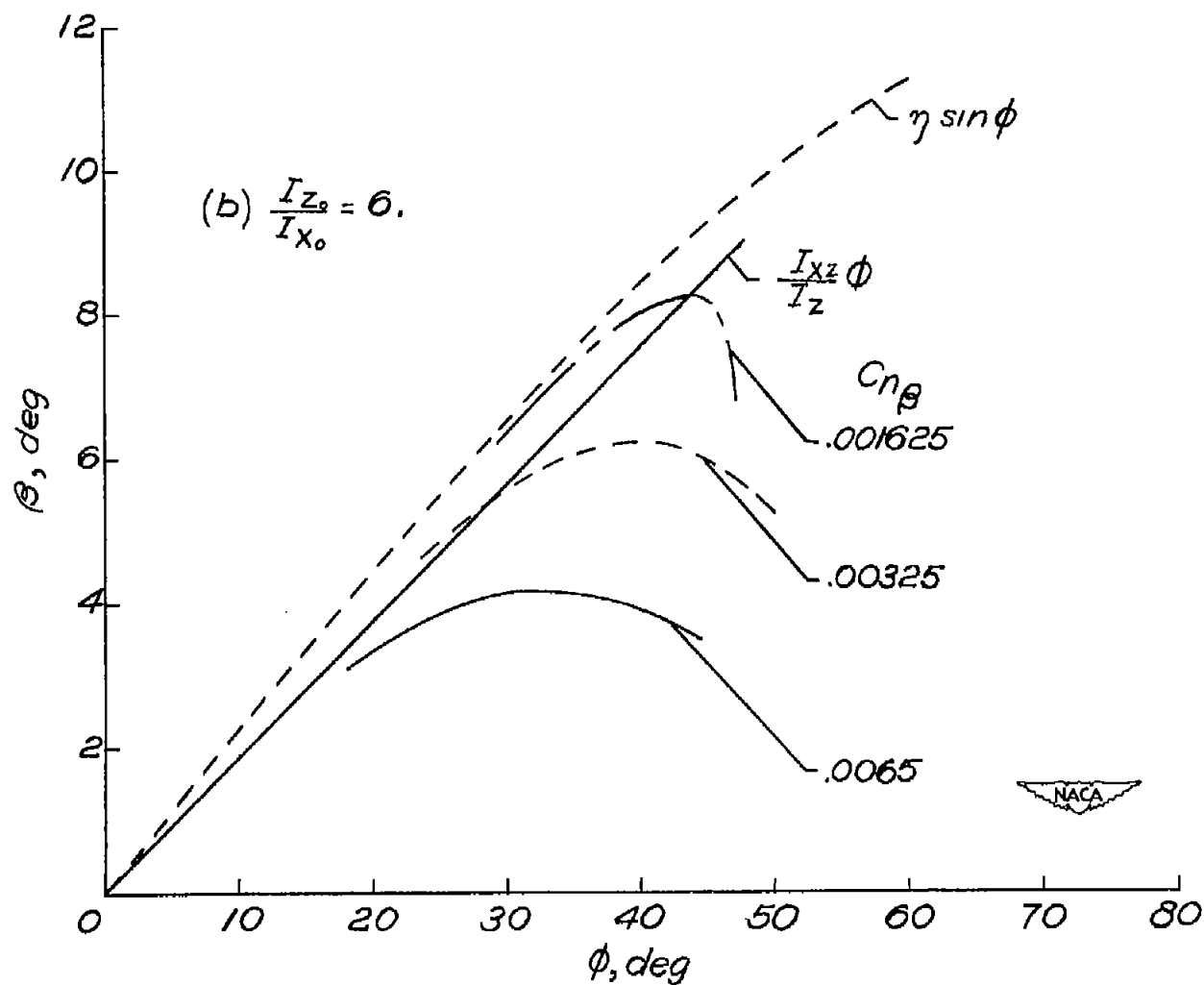


Figure 8.- Continued.

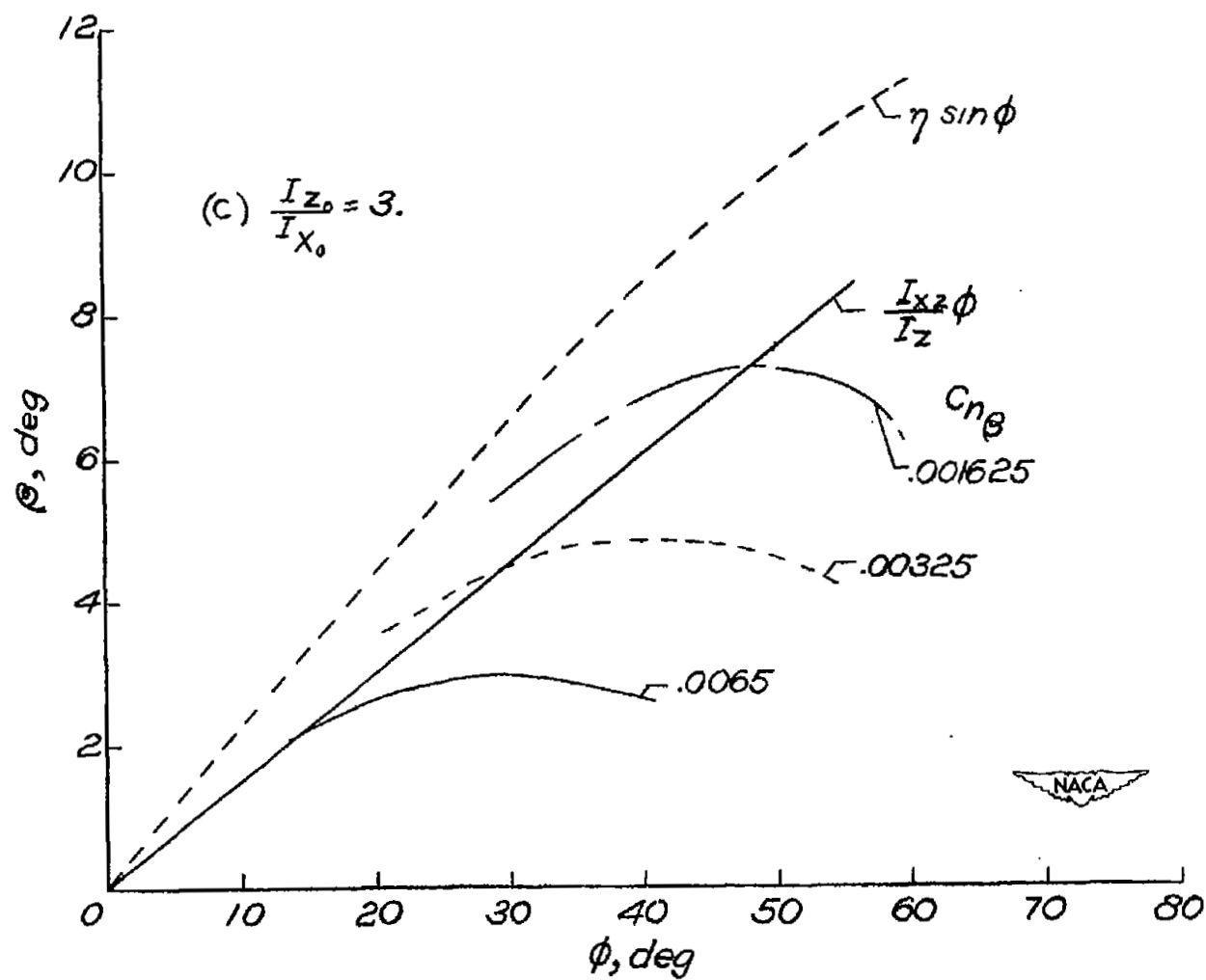


Figure 8.- Concluded.

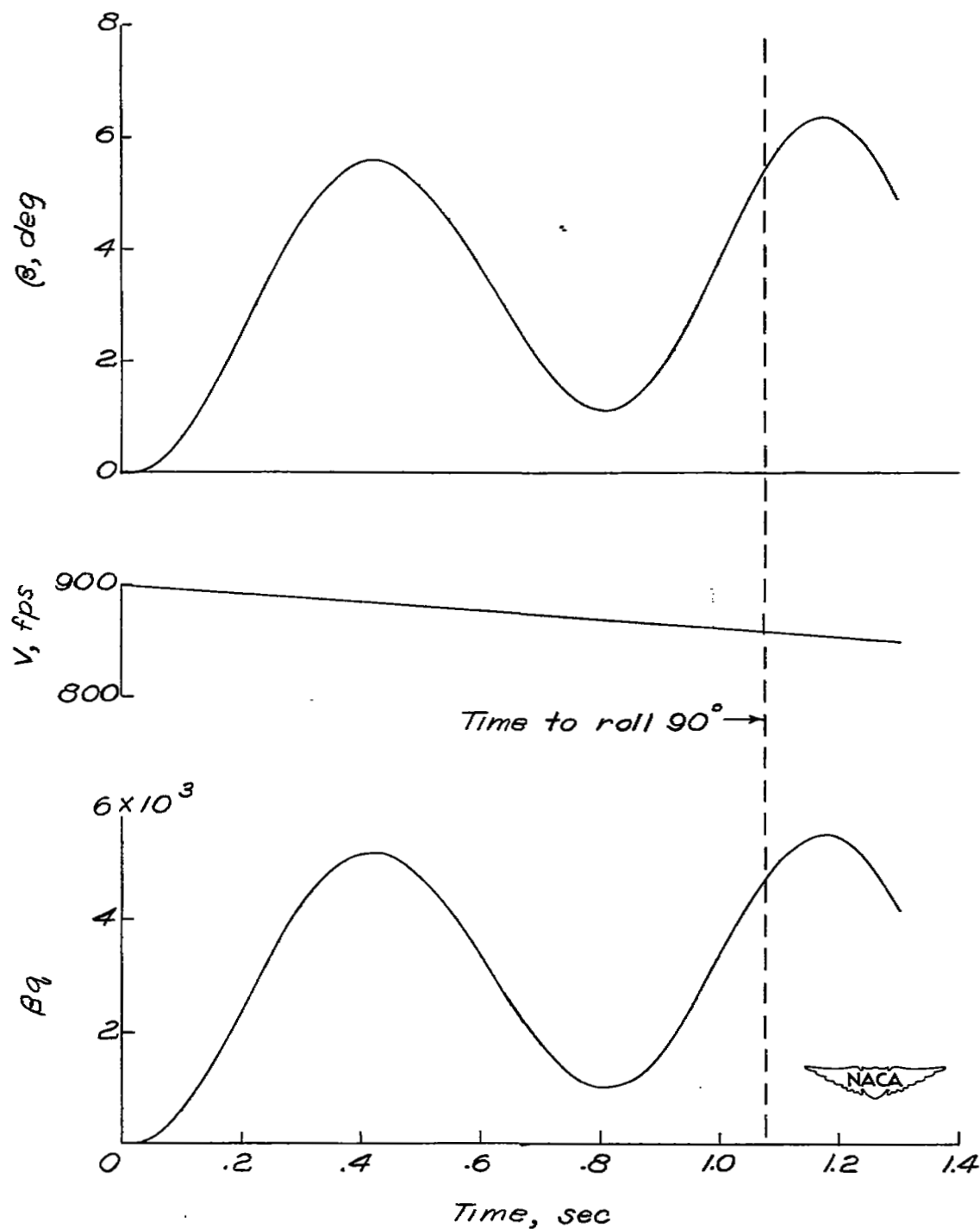


Figure 9.- Oscillations of the sideslip angle and βq (proportional to the vertical-tail load) and a probable variation in velocity in a rolling pullout with fixed elevator position. $q = \frac{1}{2}\rho V^2$.

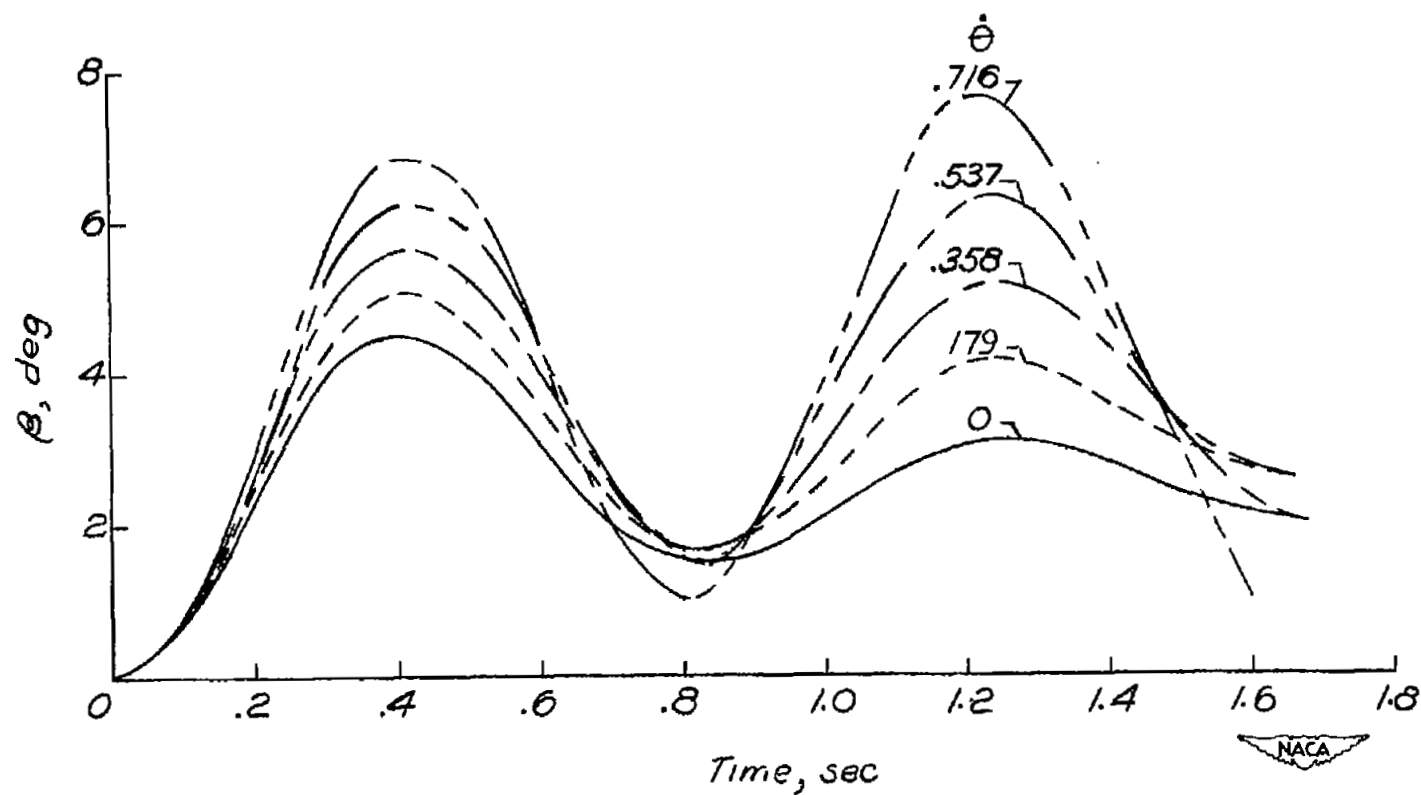
(a) β .

Figure 10.- Effects of pitching velocity on the variations of sideslip angle with time. Airplane with basic derivatives and $\frac{I_{Z_0}}{I_{X_0}} = 12$.

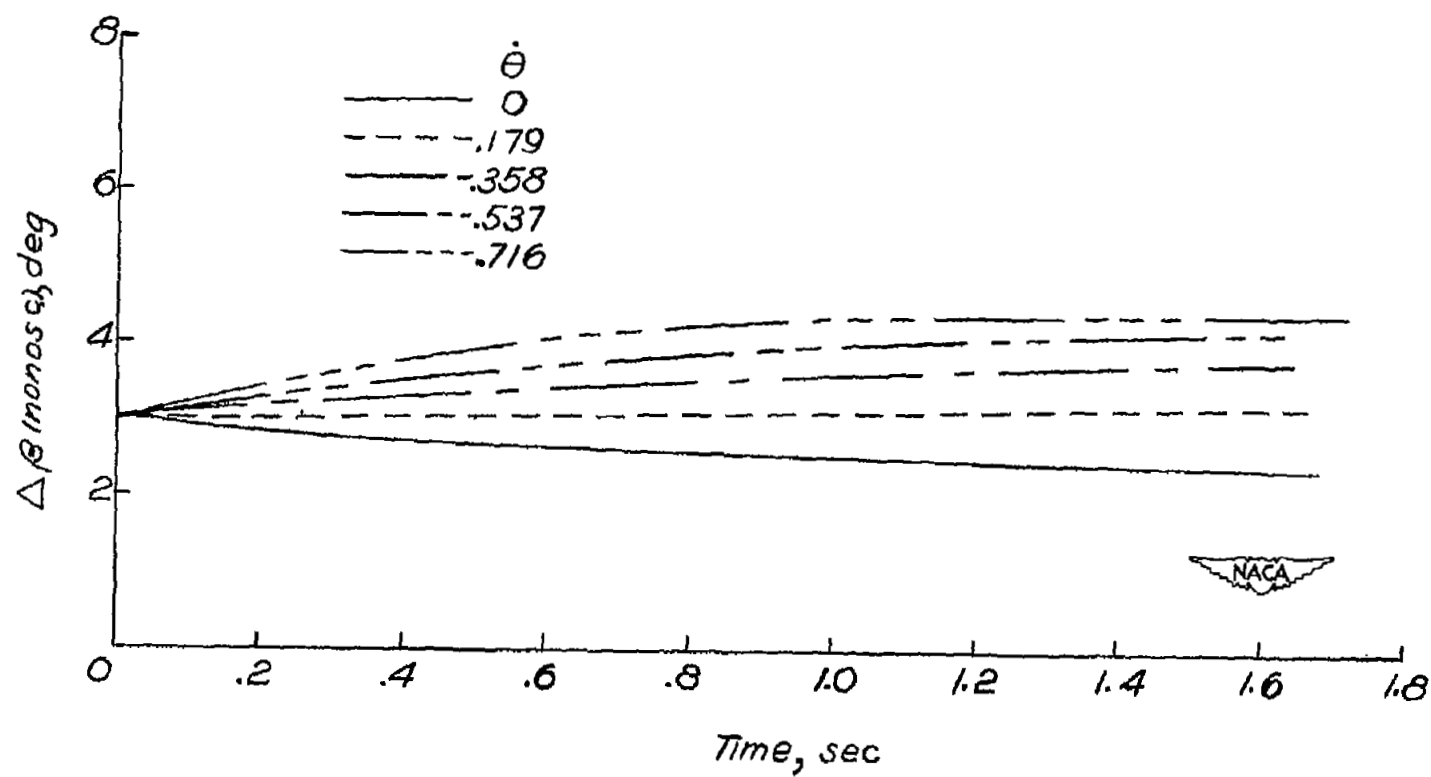
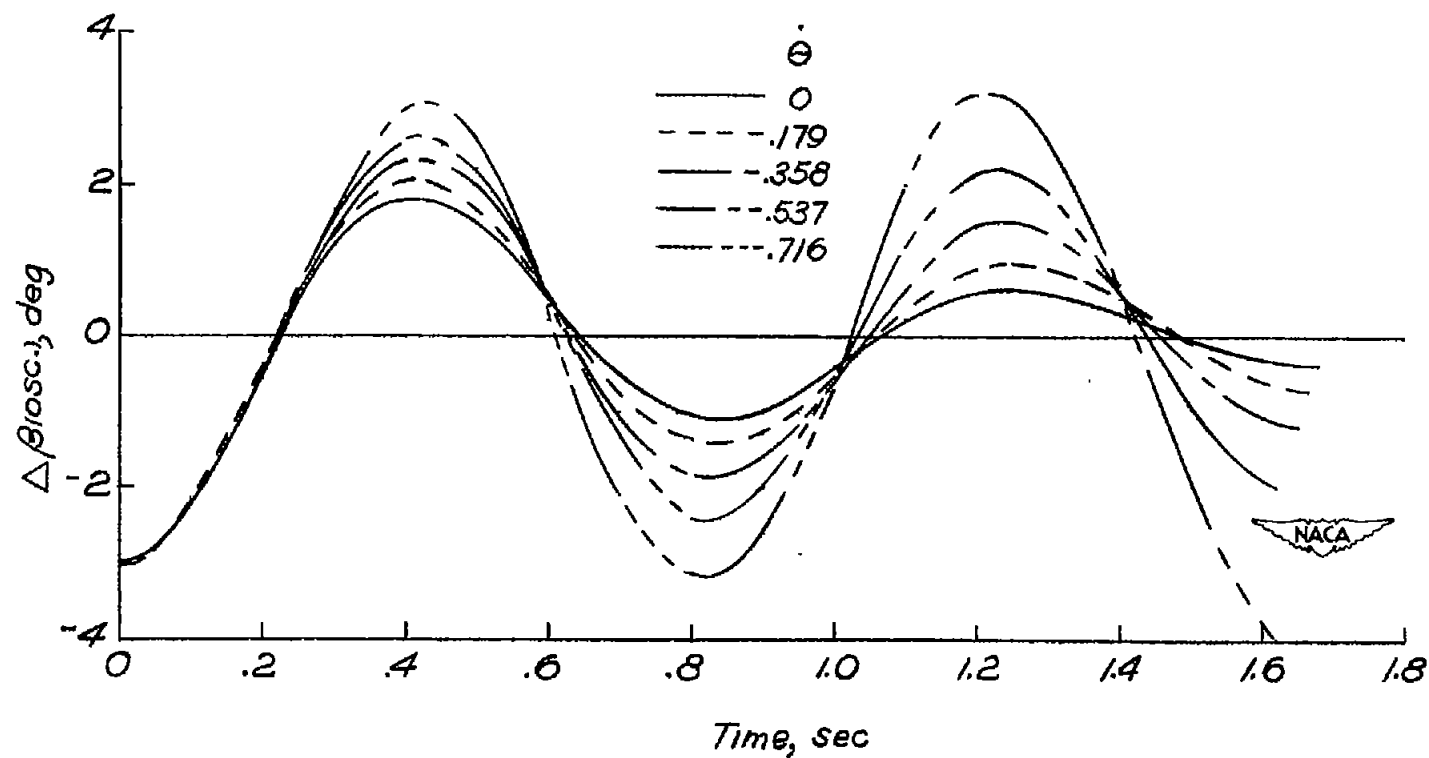
(b) Nonoscillatory increment of β

Figure 10.- Continued.



(c) Oscillatory increment of β .

Figure 10.- Concluded.

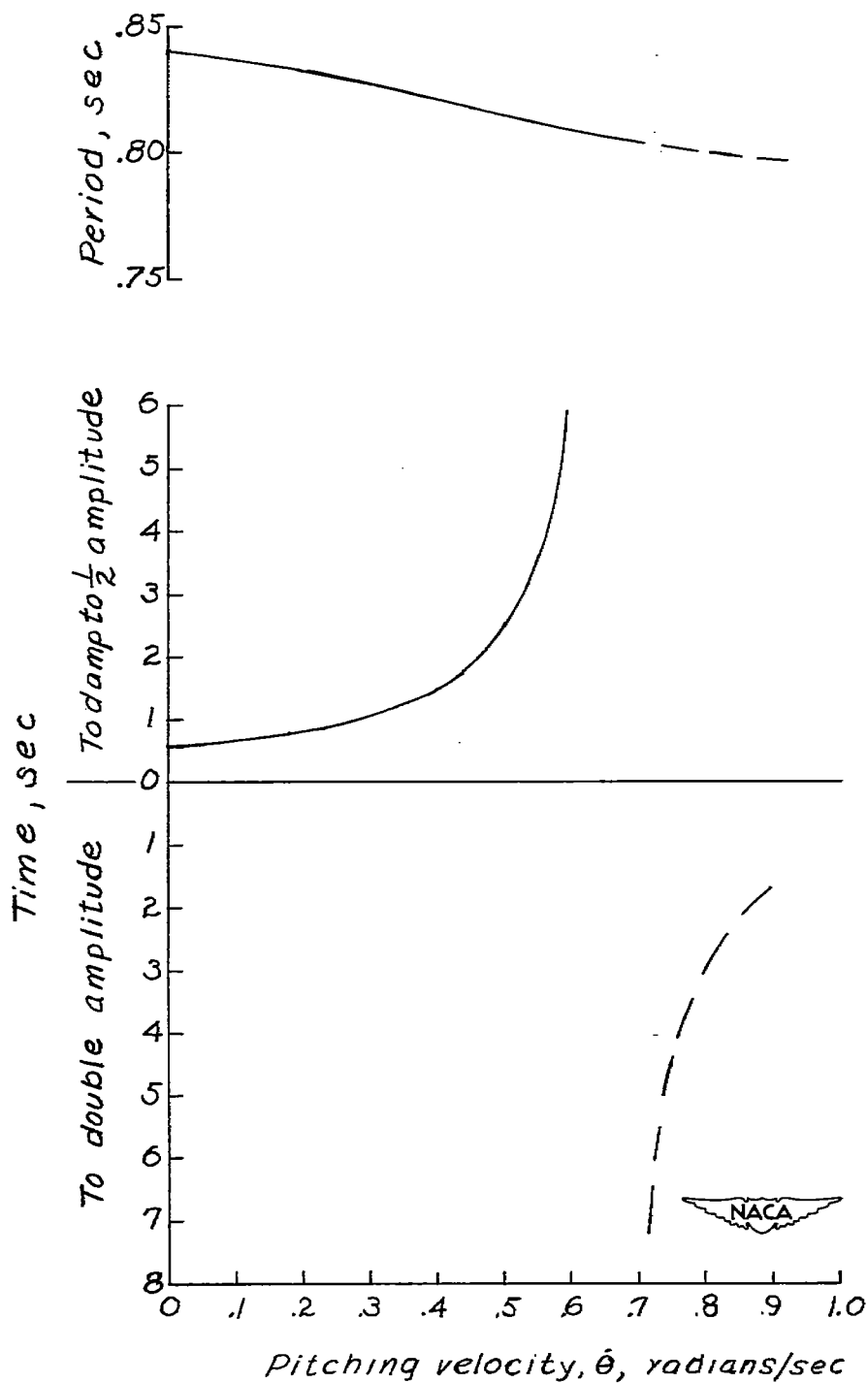


Figure 11.- Effect of pitching velocity on lateral oscillatory stability.

Airplane with basic derivatives and $\frac{I_{Z_0}}{I_{X_0}} = 12.$

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